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Water Movements in lakes during Summer Stratification; Evidence from the Distribution of Temperature in Windermere: Appendix: Oscillations in a Three-Layered Stratified Basin

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Appendix. Oscillations in a three-layered stratified basin*

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Consider a rectangular basin of length l and depth h which, in equilibrium, contains three horizontal layers of fluid of different densities ρ_1, ρ_2 and ρ_3 and of depths h_1, h_2 and h_3. (The suffixes 1, 2 and 3 refer to the upper, middle and lower layers respectively.) Let rectangular co-ordinates (x, y, z) be taken with the origin in the upper surface at one end of the basin, the x-axis horizontal and the z-axis vertically downwards. The motion will be assumed to be two-dimensional and independent of the y-co-ordinate.

Since the wave-length of the motion to be considered will be large compared with the depth, the vertical acceleration may be neglected, so that the pressure equals the hydrostatic pressure. Thus, if ζ_1, ζ_2 and ζ_3 denote the downward displacement of the upper surfaces of the three layers from their equilibrium positions, the pressures p_1, p_2 and p_3 in the three layers are given by

$$\left.\begin{aligned} p_1 &= g\rho_1(z-\zeta_1),\\ p_2 &= g\rho_1(h_1+\zeta_2-\zeta_1)+g\rho_2(z-h_1-\zeta_2),\\ p_3 &= g\rho_1(h_1+\zeta_2-\zeta_1)+g\rho_2(h_2+\zeta_3-h_1-\zeta_2)+g\rho_3(z-h_2-\zeta_3).\end{aligned}\right\} \tag{A 1}$$

If u_1, u_2 and u_3 denote the horizontal velocities, and if squares of the velocity are neglected, we have

$$\frac{\mathrm{d}u_1}{\mathrm{d}t} = -\frac{1}{\rho_1}\frac{\mathrm{d}p_1}{\mathrm{d}x}, \quad \frac{\mathrm{d}u_2}{\mathrm{d}t} = -\frac{1}{\rho_2}\frac{\mathrm{d}p_2}{\mathrm{d}x}, \quad \frac{\mathrm{d}u_3}{\mathrm{d}t} = -\frac{1}{\rho_3}\frac{\mathrm{d}p_3}{\mathrm{d}x}, \tag{A 2}$$

which shows that u_1, u_2 and u_3 are independent of z, i.e. that the motion in any vertical plane is uniform for each layer. Thirdly, by considering the flux of water into any element of volume we obtain the equations of continuity:

$$\frac{\mathrm{d}}{\mathrm{d}x}(h_1u_1) = \frac{\mathrm{d}}{\mathrm{d}t}(\zeta_1-\zeta_2), \quad \frac{\mathrm{d}}{\mathrm{d}x}(h_2u_2) = \frac{\mathrm{d}}{\mathrm{d}t}(\zeta_2-\zeta_3), \quad \frac{\mathrm{d}}{\mathrm{d}x}h_3u_3 = \frac{\mathrm{d}\zeta_3}{\mathrm{d}t}. \tag{A 3}$$

On eliminating p_1, p_2, p_3, u_1, u_2 and u_3 from equations (A 1, 2 and 3) we find

$$\left.\begin{aligned} \frac{\mathrm{d}^2}{\mathrm{d}t^2}(\zeta_1-\zeta_2) &= gh_1\frac{\mathrm{d}^2}{\mathrm{d}x^2}[(\zeta_1-\zeta_2)+(\zeta_2-\zeta_3)+\zeta_3],\\ \frac{\mathrm{d}^2}{\mathrm{d}t^2}(\zeta_2-\zeta_3) &= gh_1\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[\frac{\rho_1}{\rho_2}(\zeta_1-\zeta_2)+(\zeta_2-\zeta_3)+\zeta_3\right],\\ \frac{\mathrm{d}^2}{\mathrm{d}t^2}\zeta_3 &= gh_1\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[\frac{\rho_1}{\rho_3}(\zeta_1-\zeta_2)+\frac{\rho_1}{\rho_2}(\zeta_2-\zeta_3)+\zeta_3\right].\end{aligned}\right\} \tag{A 4}$$

We seek a simple harmonic motion of wave-length $2\pi/k$ and period $2\pi/\sigma$; the operators $\mathrm{d}^2/\mathrm{d}t^2$ and $\mathrm{d}^2/\mathrm{d}x^2$ are therefore to be replaced by $-\sigma^2$ and $-k^2$ respectively, and we have

$$\left.\begin{aligned} (h_1+H)(\zeta_1-\zeta_2)+h_1(\zeta_2-\zeta_3)+h_1\zeta_3 &= 0,\\ h_2\frac{\rho_1}{\rho_2}(\zeta_1-\zeta_2)+(h_2+H)(\zeta_2-\zeta_3)+h_2\zeta_3 &= 0,\\ h_3\frac{\rho_1}{\rho_3}(\zeta_1-\zeta_2)+h_3\frac{\rho_2}{\rho_3}(\zeta_2-\zeta_3)+(h_3+H)\zeta_3 &= 0,\end{aligned}\right\} \tag{A 5}$$

where
$$H = -\sigma^2/gk^2. \tag{A 6}$$

* Some of the following results have been obtained by Makkaweev (1936) and other writers; they are collected here for convenience of reference.

The elimination of $(\zeta_1-\zeta_2)$, $(\zeta_2-\zeta_3)$ and ζ_3 from (A 5) gives

$$\begin{vmatrix} 1+\dfrac{H}{h_1} & 1 & 1 \\ \dfrac{\rho_1}{\rho_2} & 1+\dfrac{H}{h_2} & 1 \\ \dfrac{\rho_1}{\rho_3} & \dfrac{\rho_2}{\rho_3} & 1+\dfrac{H}{h_3} \end{vmatrix} = 0, \tag{A 7}$$

which on expansion becomes

$$\begin{aligned} H^3+H^2(h_1+h_2+h_3) \\ -H\left[h_2h_3\left(\frac{\rho_2}{\rho_3}-1\right)+h_1h_3\left(\frac{\rho_1}{\rho_3}-1\right)+h_1h_2\left(\frac{\rho_1}{\rho_2}-1\right)\right] \\ +h_1h_2h_3\left(\frac{\rho_1}{\rho_2}-1\right)\left(\frac{\rho_2}{\rho_3}-1\right)=0. \end{aligned} \tag{A 8}$$

Equation (A 8) determines the relation between the period and the wave-length. Since the left-hand side is a cubic in H, there will be, for a given wave-length, three possible modes of oscillation which we may denote by the indices (1), (2) and (3). For each value of $H^{(i)}$ the ratios of the corresponding displacements $\zeta_1^{(i)}$, $\zeta_2^{(i)}$ and $\zeta_3^{(i)}$ may be found from equations (A 5). Thus we have

$$\zeta_1^{(i)}:\zeta_2^{(i)}:\zeta_3^{(i)} = H^{(i)}:h_1+H^{(i)}:\frac{H^{(i)}(h_1+H^{(i)})\,h_3\rho_2}{h_2h_3(\rho_3-\rho_2)+H^{(i)}(h_2\rho_3+h_3\rho_2)}. \tag{A 9}$$

The density differences recurring in practice are of the order of one part per thousand. Thus one solution of (8) will be given very nearly by

$$H^{(1)} = -(h_1+h_2+h_3),$$

that is,

$$\frac{\sigma^{(1)2}}{k^2} = gh. \tag{A 10}$$

This is the ordinary 'surface' seiche. From (A 9) we have then

$$\zeta_1^{(1)}:\zeta_2^{(1)}:\zeta_3^{(1)} = h_1+h_2+h_3:h_2+h_3:h_3, \tag{A 11}$$

showing that the displacements are all in the same sense and are proportional to the height above the bottom. The two 'internal' seiches, in which we are chiefly interested here, are given by the remaining roots $H^{(2)}$ and $H^{(3)}$ of equation (A 5). Since these roots are of the order of (ρ_2/ρ_3-1) they satisfy the equation

$$\begin{aligned} H^2(h_1+h_2+h_3) \\ -H\left[h_2h_3\left(\frac{\rho_2}{\rho_3}-1\right)+h_1h_3\left(\frac{\rho_1}{\rho_3}-1\right)+h_1h_2\left(\frac{\rho_1}{\rho_2}-1\right)\right] \\ +h_1h_2h_3\left(\frac{\rho_1}{\rho_2}-1\right)\left(\frac{\rho_2}{\rho_3}-1\right)=0 \end{aligned} \tag{A 12}$$

approximately (the term in H^3 in equation (A 5) being now negligible). The ratios of the corresponding displacements are found from (9):

$$\zeta_1^{(i)}:\zeta_2^{(i)}:\zeta_3^{(i)} = H^{(i)}:h_1:\frac{H^{(i)}h_1h_3}{h_2h_3(\rho_3/\rho_2-1)+H^{(i)}(h_2+h_3)} \tag{A 13}$$

($i = 2, 3$) to the same order of approximation. Thus in the two 'internal' seiches the displacement of the free surface is very small compared with that of the two interfaces. It will be convenient to denote the ratio of the other two displacements by $\beta^{(i)}$:

$$\beta^{(i)} = \zeta_2^{(i)}/\zeta_3^{(i)} = (h_2/H^{(i)})\,(\rho_3/\rho_2 - 1) + (h_2/h_3 + 1). \tag{A 14}$$

Since there is no flow across the vertical walls $x = 0, l$, l must be a multiple of half a wave-length; hence

$$\left.\begin{aligned} k &= n\pi/l = nk_0,\\ \sigma^{(i)} &= k\sqrt{\{-gH^{(i)}\}} = n\sigma_0^{(i)},\end{aligned}\right\} \tag{A 15}$$

where

$$k_0 = \pi/l, \quad \sigma_0^{(i)} = (\pi/l)\sqrt{\{-gH^{(i)}\}}. \tag{A 16}$$

In general the motion will consist of the sum of an infinite number of modes whose wave-length is given by (A 15). If the initial displacement of the free surface is small compared with that of the two interfaces, the modes of 'surface' type (corresponding to $H = H^{(1)}$) will not be excited. Hence we shall have

$$\left.\begin{aligned} \zeta_1 &= 0,\\ \zeta_2 &= \sum_{n=1}^{\infty} \cos nk_0 x(\beta^{(2)} A_n^{(2)} \cos n\sigma_0^{(2)} t + \beta^{(2)} B_n^{(2)} \sin n\sigma_0^{(2)} t)\\ &\quad + \sum_{n=1}^{\infty} \cos nk_0 x(\beta^{(3)} A_n^{(3)} \cos n\sigma_0^{(3)} t + \beta^{(3)} B_n^{(3)} \sin n\sigma_0^{(3)} t),\\ \zeta_3 &= \sum_{n=1}^{\infty} \cos nk_0 x(A_n^{(2)} \cos n\sigma_0^{(2)} t + B_n^{(2)} \sin n\sigma_0^{(2)} t)\\ &\quad + \sum_{n-1}^{\infty} \cos nk_0 x(A_n^{(3)} \cos n\sigma_0^{(3)} t + B_n^{(3)} \sin n\sigma_0^{(3)} t),\end{aligned}\right\} \tag{A 17}$$

where $A_n^{(2)}$, $A_n^{(3)}$, $B_n^{(2)}$ and $B_n^{(3)}$ are constants to be determined from the initial conditions. Suppose that when $t = 0$ we have

$$\left.\begin{aligned} \zeta_2 &= f_2(x), \quad \zeta_3 = f_3(x),\\ \frac{d\zeta_2}{dt} &= g_2(x), \quad \frac{d\zeta_3}{dt} = g_3(x).\end{aligned}\right\} \tag{A 18}$$

Then on writing $t = 0$ in (A 17), we have

$$\left.\begin{aligned} \Sigma(\beta^{(2)} A_n^{(2)} + \beta^{(3)} A_n^{(3)}) \cos nk_0 x &= f_2(x),\\ \Sigma(A_n^{(2)} + A_n^{(3)}) \cos nk_0 x &= f_3(x),\end{aligned}\right\} \tag{A 19}$$

and

$$\left.\begin{aligned} \Sigma(n\sigma_0^{(2)} \beta^{(2)} B_n^{(2)} + n\sigma_0^{(3)} \beta^{(3)} B_n^{(3)}) \cos nk_0 x &= g_2(x),\\ \Sigma(n\sigma_0^{(2)} B_n^{(2)} + n\sigma_0^{(3)} B_n^{(3)}) \cos nk_0 x &= g_3(x).\end{aligned}\right\} \tag{A 20}$$

By Fourier's theorem, equation (A 19) can be satisfied provided

$$\left.\begin{aligned} \beta^{(2)} A_n^{(2)} + \beta^{(3)} A_n^{(3)} &= \frac{2}{l}\int_0^l f_2(x) \cos nk_0 x\,dx,\\ A_n^{(2)} + A_n^{(3)} &= \frac{2}{l}\int_0^l f_3(x) \cos nk_0 x\,dx,\end{aligned}\right\} \tag{A 21}$$

and so

$$\left.\begin{aligned} A_n^{(2)} &= \frac{2}{l}\int_0^l \frac{f_2(x)-\beta^{(3)}f_3(x)}{\beta^{(2)}-\beta^{(3)}}\cos nk_0 x\,dx, \\ A_n^{(3)} &= \frac{2}{l}\int_0^l \frac{f_2(x)-\beta^{(2)}f_3(x)}{\beta^{(3)}-\beta^{(2)}}\cos nk_0 x\,dx. \end{aligned}\right\} \tag{A 22}$$

Similarly,

$$\left.\begin{aligned} n\sigma^{(2)}B_n^{(2)} &= \frac{2}{l}\int_0^l \frac{g_2(x)-\beta^{(3)}g_3(x)}{\beta^{(2)}-\beta^{(3)}}\cos nk_0 x\,dx, \\ n\sigma^{(3)}B_n^{(3)} &= \frac{2}{l}\int_0^l \frac{g_2(x)-\beta^{(2)}g_3(x)}{\beta^{(3)}-\beta^{(2)}}\cos nk_0 x\,dx. \end{aligned}\right\} \tag{A 23}$$

When the motion starts from rest, it is clear that $B_n^{(2)}$ and $B_n^{(3)}$ are zero.

The coefficients $A_n^{(2)}$, $A_n^{(3)}$, $B_n^{(2)}$ and $B_n^{(3)}$ being found from (A 22) and (A 23), the complete solution is now given by (A 17).

The horizontal velocities (u_1, u_2 and u_3 for the top, middle and bottom layers respectively) are found from the equations of continuity (equation (A 3)). These give

$$\left.\begin{aligned} h_1 u_1 = &\sum_{n=1}^{\infty}\frac{\sigma_0^{(2)}}{k_0}\sin nk_0 x(\beta^{(2)}A_n^{(2)}\sin n\sigma_0^{(2)}t-\beta^{(2)}B_n^{(2)}\cos n\sigma_0^{(2)}t) \\ &+\sum_{n=1}^{\infty}\frac{\sigma_0^{(3)}}{k_0}\sin nk_0 x(\beta^{(3)}A_n^{(3)}\sin n\sigma_0^{(3)}t-\beta^{(3)}B_n^{(3)}\cos n\sigma_0^{(3)}t), \end{aligned}\right\} \tag{A 24}$$

$$\left.\begin{aligned} -h_3 u_3 = &\sum_{n=1}^{\infty}\frac{\sigma_0^{(2)}}{k_0}\sin nk_0 x(A_n^{(2)}\sin n\sigma_0^{(2)}t-B_n^{(2)}\cos n\sigma_0^{(2)}t) \\ &+\sum_{n=1}^{\infty}\frac{\sigma_0^{(3)}}{k_0}\sin nk_0 x(A_n^{(3)}\sin n\sigma_0^{(3)}t-B_n^{(3)}\cos n\sigma_0^{(3)}t), \end{aligned}\right\} \tag{A 25}$$

and

$$h_2 u_2 = -(h_1 u_1 + h_3 u_3). \tag{A 26}$$

In the plane $x = \frac{1}{2}l$, and when the motion initially starts from rest, we have

$$\left.\begin{aligned} h_1 u_1 = &\sum_{n=1}^{\infty}\frac{\sigma_0^{(2)}}{k_0}\sin\tfrac{1}{2}n\pi\,.\,\beta^{(2)}A_n^{(2)}\sin n\sigma_0^{(2)}t \\ &+\sum_{n=0}^{\infty}\frac{\sigma_0^{(3)}}{k_0}\sin\tfrac{1}{2}n\pi\,.\,\beta^{(3)}A_n^{(3)}\sin n\sigma_0^{(3)}t, \end{aligned}\right\} \tag{A 27}$$

and

$$\left.\begin{aligned} -h_3 u_3 = &\sum_{n=1}^{\infty}\frac{\sigma_0^{(2)}}{k_0}\sin\tfrac{1}{2}n\pi\,.\,A_n^{(2)}\sin n\sigma_0^{(2)}t \\ &+\sum_{n=1}^{\infty}\frac{\sigma_0^{(3)}}{k_0}\sin\tfrac{1}{2}n\pi\,.\,A_n^{(3)}\sin n\sigma_0^{(3)}t. \end{aligned}\right\} \tag{A 82}$$

u_2 may be found from equation (A 26) as in the general case.

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